RC4200
Analog Multiplier

Features
- High accuracy
- Nonlinearity – 0.1%
  Temperature coefficient – 0.005%/°C
- Multiple functions
- Multiply, divide, square, square root, RMS-to-DC conversion, AGC and modulate/demodulate
- Wide bandwidth – 4 MHz
- Signal-to-noise ratio – 94 dB

Description
The RC4200 analog multiplier has complete compensation for nonlinearity, the primary source of error and distortion. This multiplier also has three onboard operational amplifiers designed specifically for use in multiplier logging circuits. These amplifiers are frequency compensated for optimum AC response in a logging circuit, the heart of a multiplier, and can therefore provide superior AC response.

The RC4200 can be used in a wide variety of applications without sacrificing accuracy. Four-quadrant multiplication, two-quadrant division, square rooting, squaring and RMS conversion can all be easily implemented with predictable accuracy. The nonlinearity compensation is not just trimmed at a single temperature, it is designed to provide compensation over the full temperature range. This nonlinearity compensation combined with the low gain and offset drift inherent in a well-designed monolithic chip provides a very high accuracy and a low temperature coefficient.

The RC4200 is ideal for use in low distortion audio modulation circuits, voltage-controlled active filters, and precision oscillators.

Block Diagram

![Block Diagram of RC4200 Analog Multiplier](image-url)
**Functional Description**

The RC4200 multiplier is designed to multiply two input currents (I1 and I2) and to divide by a third input current (I4). The output is also in the form of a current (I3). A simplified circuit diagram is shown in the Block Diagram. The nominal relationship between the three inputs and the output is:

\[
I_3 = \frac{I_1 I_2}{I_4} \quad (1)
\]

The three input currents must be positive and restricted to a range of 1 μA to 1 mA. These currents go into the multiplier chip at op amp summing junctions which are nominally at zero volts. Therefore, an input voltage can be easily converted to an input current by a series resistor. Any number of currents may be summed at the inputs. Depending on the application, the output current can be converted to a voltage by an external op amp or used directly. This capability of combining input currents and voltages in various combinations provides great versatility in application.

Inside the multiplier chip, the three op amps make the collector currents of transistors Q1, Q2 and Q4 equal to their respective input currents (I1, I2, and I4). These op amps are designed with current source outputs and are phase-compensated for optimum frequency response as a multiplier. Power drain of the op amps was minimized to prevent the introduction of undesired thermal gradients on the chip. The three op amps operate on a single supply voltage (nominally -15V) and total quiescent current drain is less than 4 mA. These special op amps provide significantly improved performance in comparison to 741-type op amps.

The actual multiplication is done within the log-antilog configuration of the Q1-Q4 transistor array. These four transistors, with associated proprietary circuitry, were specially designed to precisely implement the relationship.

\[
V_{BEH} = \frac{KT}{Q} \ln \frac{I_{CN}}{I_{SN}} \quad (2)
\]

Previous multiplier designs have suffered from an additional undesired linear term in the above equation; the collector current times the emitter resistance. The I<sub>CE</sub> term introduces a parabolic nonlinearity even with matched transistors. Raytheon has developed a unique and proprietary means of inherently compensating for this undesired I<sub>CE</sub> term. Furthermore, this Raytheon developed circuit technique compensates linearity error over temperature changes. The nonlinearity versus temperature is significantly improved over earlier designs.

From equation (2) and by assuming equal transistor junction temperatures, summing base-to-emitter voltage drops around the transistor array yields:

\[
\frac{KT}{q} \left( \ln \frac{I_1}{I_{S1}} - \ln \frac{I_2}{I_{S2}} - \ln \frac{I_3}{I_{S3}} - \ln \frac{I_4}{I_{S4}} \right) = 0 \quad (3)
\]

This equation reduces to:

\[
\frac{I_1 I_2}{I_3 I_4} = \frac{I_{S1} I_{S2}}{I_{S3} I_{S4}} \quad (4)
\]

The rate of reverse saturation current I<sub>S1</sub>I<sub>S2</sub>/I<sub>S3</sub>I<sub>S4</sub>, depends on the transistor matching. In a monolithic multiplier this matching is easily achieved and the rate is very close to unity, typically 1.0±1%. The final result is the desired relationship:

\[
I_3 = \frac{I_1 I_2}{I_4} \quad (5)
\]

The inherent linearity and gain stability combined with low cost and versatility makes this new circuit ideal for a wide range of nonlinear functions.

**Pin Assignments**

![Pin Diagram](image-url)
### Absolute Maximum Ratings

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Min</th>
<th>Max</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply Voltage (^1)</td>
<td>-22</td>
<td></td>
<td>V</td>
</tr>
<tr>
<td>Input Current</td>
<td>-5</td>
<td></td>
<td>mA</td>
</tr>
<tr>
<td>Storage Temperature Range</td>
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<td></td>
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<tr>
<td>RM4200/4200A</td>
<td>-65</td>
<td>+150</td>
<td>°C</td>
</tr>
<tr>
<td>RC4200/4200A</td>
<td>-55</td>
<td>+125</td>
<td>°C</td>
</tr>
<tr>
<td>Operating Temperature Range</td>
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<td></td>
<td></td>
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<tr>
<td>RM4200/4200A</td>
<td>-55</td>
<td>+125</td>
<td>°C</td>
</tr>
<tr>
<td>RC4200/4200A</td>
<td>0</td>
<td>+70</td>
<td>°C</td>
</tr>
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</table>

**Notes:**
1. For a supply voltage greater than -22V, the absolute maximum input voltage is equal to the supply voltage.
2. Observe package thermal characteristics.

### Thermal Characteristics

(Still air, soldered into PC board)

<table>
<thead>
<tr>
<th></th>
<th>8-Lead Plastic DIP</th>
<th>8-Lead Ceramic DIP</th>
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</thead>
<tbody>
<tr>
<td>Max. Junction Temp.</td>
<td>+125°C</td>
<td>+175°C</td>
</tr>
<tr>
<td>Max. Pd T&lt;sub&gt;A&lt;/sub&gt;&lt;50°C</td>
<td>468 mW</td>
<td>833 mW</td>
</tr>
<tr>
<td>Therm. Res θJC</td>
<td>—</td>
<td>45°C/W</td>
</tr>
<tr>
<td>Therm. Res. θJA</td>
<td>160°C/W</td>
<td>150°C/W</td>
</tr>
<tr>
<td>For T&lt;sub&gt;A&lt;/sub&gt; &gt; 50°C Derate at</td>
<td>6.25 mW/°C</td>
<td>8.33 mW/°C</td>
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</tbody>
</table>

### Electrical Characteristics

(Over operating temperature range, V<sub>S</sub> = -15V unless otherwise noted)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Test Conditions</th>
<th></th>
<th>4200A</th>
<th></th>
<th>4200</th>
<th>Units</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>Min</td>
<td>Typ</td>
<td>Max.</td>
<td>Min</td>
<td>Typ</td>
</tr>
<tr>
<td>Total Error as Multiplier Untrimmed</td>
<td>TA = +25°C</td>
<td>±2.0</td>
<td>±0.2</td>
<td>±0.05</td>
<td>±0.2</td>
<td>±0.05</td>
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<tr>
<td>With External Trim Versus Temperature</td>
<td></td>
<td>±0.1</td>
<td>±0.05</td>
<td>±0.05</td>
<td>±0.1</td>
<td>±0.05</td>
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<tr>
<td>Versus Supply (-9 to -18V)</td>
<td></td>
<td>±0.1</td>
<td>±0.1</td>
<td>±0.1</td>
<td>±0.3</td>
<td>±0.3</td>
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<tr>
<td>Nonlinearity</td>
<td>50µA ≤ I&lt;sub&gt;1,4&lt;/sub&gt; ≤ 250 µA, TA = +25°C (Note 2)</td>
<td>±0.1</td>
<td>±0.1</td>
<td>±0.1</td>
<td>±0.3</td>
<td>±0.3</td>
</tr>
<tr>
<td>Input Current Range (I&lt;sub&gt;1,2&lt;/sub&gt; and I&lt;sub&gt;4&lt;/sub&gt;)</td>
<td>1.0</td>
<td>1000</td>
<td>1.0</td>
<td>1000</td>
<td>µA</td>
<td></td>
</tr>
<tr>
<td>Input Offset Voltage</td>
<td>I&lt;sub&gt;1&lt;/sub&gt; = I&lt;sub&gt;2&lt;/sub&gt; = I&lt;sub&gt;4&lt;/sub&gt; = 150 µA, TA = +25°C</td>
<td>±5.0</td>
<td>±5.0</td>
<td>±10</td>
<td>mV</td>
<td></td>
</tr>
<tr>
<td>Input Bias Current</td>
<td>I&lt;sub&gt;1&lt;/sub&gt; = I&lt;sub&gt;2&lt;/sub&gt; = I&lt;sub&gt;4&lt;/sub&gt; = 150 µA, TA = +25°C</td>
<td>300</td>
<td>300</td>
<td>500</td>
<td>nA</td>
<td></td>
</tr>
<tr>
<td>Average Input Offset Voltage Drift</td>
<td>I&lt;sub&gt;1&lt;/sub&gt; = I&lt;sub&gt;2&lt;/sub&gt; = I&lt;sub&gt;4&lt;/sub&gt; = 150 µA</td>
<td>±50</td>
<td>±50</td>
<td>±100</td>
<td>µV/°C</td>
<td></td>
</tr>
<tr>
<td>Output Current Range (I3)</td>
<td>Note 3</td>
<td>1.0</td>
<td>1000</td>
<td>1.0</td>
<td>1000</td>
<td>µA</td>
</tr>
</tbody>
</table>
Electrical Characteristics (continued)
(Over operating temperature range, $V_S = -15\,\text{V}$ unless otherwise noted)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Test Conditions</th>
<th>4200A</th>
<th>4200</th>
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<tr>
<td>Frequency Response,</td>
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<td>4.0</td>
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<tr>
<td>-3dB point</td>
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<td>-18</td>
<td>-18</td>
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<tr>
<td>Supply Voltage</td>
<td></td>
<td>-15</td>
<td>-15</td>
</tr>
<tr>
<td>Supply Current</td>
<td>$I_1 = I_2 = I_4 = 150,\mu\text{A}$</td>
<td>4.0</td>
<td>4.0</td>
</tr>
<tr>
<td>$T_A = +25^\circ\text{C}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:
1. Refer to Figure 6 for example.
2. The input circuits tend to become unstable at $I_1$, $I_2$, $I_4 < 50\,\mu\text{A}$ and linearity decreases when $I_1$, $I_2$, $I_4 > 250\,\mu\text{A}$ (eq. @ $I_1 = I_2 = 500\,\mu\text{A}$, nonlinearity error = 0.5%).
3. These specifications apply with output (b) connected to an op amp summing junction. If desired, the output (b) at pin (4) can be used to drive a resistive load directly. The resistive load should be less than 700Ω and must be pulled up to a positive supply such that the voltage on pin (4) stays within a range of 0 to +5V.

Basic Circuits

Current Multiplier/Divider
The basic design criteria for all circuit configurations using the 4200 multiplier is contained in equation (1): i.e.,

$$I_3 = \frac{I_1 I_2}{I_4}$$

The current-product-balance equation restates this as:

$$I_1 I_2 = I_3 I_4 \quad (6)$$

Dynamic Range and Stability
The precision dynamic range for the 4200 is from $+50\,\mu\text{A}$ to $+250\,\mu\text{A}$ inputs for $I_1$, $I_2$ and $I_4$. Stability and accuracy degrade if this range is exceeded.

To improve the stability for input currents less than $50\,\mu\text{A}$, filter circuits ($R_S C_S$) are added to each input (see Figure 2).

![Figure 1](image1.png)

**Figure 1.**
Amplifier A1 is used to convert the $I_3$ current to an output voltage.

Multiplier: $V_Z = \text{constant} \neq 0$
Divider: $V_Y = \text{constant} \neq 0$

![Figure 2](image2.png)
Voltage Multiplier/Divider

\[ \frac{V_X V_Y}{R_1 R_2} = \frac{V_{oVZ}}{R_o R_4} = 65-4200-04 \]

**Figure 3.**

Solving for \( V_0 = \frac{V_X V_Y}{V_Z} \frac{R_0 R_4}{R_1 R_2} \)

For a multiplier circuit \( V_Z = V_R = \) constant

Therefore: \( V_0 = V_X V_Y K \) where \( K = \frac{R_0 R_4}{V_Z R_1 R_2} \)

For a divider circuit \( V_Y = V_R = \) constant

Therefore: \( V_0 = \frac{V_X}{V_Z} K \) where \( K = \frac{V_R R_0 R_4}{R_1 R_2} \)

**Extended Range**

The input and output voltage ranges can be extended to include 0 and negative voltage signals by adding bias currents. The RSCS filter circuits are eliminated when the input and biasing resistors are selected to limit the respective currents to 50 µA min. and 250 µA max.

Extended Range Multiplier

**Figure 4.**

Resistors \( R_a \) and \( R_b \) extend the range of the \( V_X \) and \( V_Y \) inputs by picking values such that:

\[ I_1 \text{ (min.)} = \frac{V_X \text{ (min.)}}{R_1} + \frac{V_{REF}}{R_a} = 50 \mu A, \]

and \( I_1 \text{ (max.)} = \frac{V_X \text{ (max.)}}{R_1} + \frac{V_{REF}}{R_a} = 250 \mu A, \)

also \( I_2 \text{ (min.)} = \frac{V_Y \text{ (min.)}}{R_2} + \frac{V_{REF}}{R_b} = 50 \mu A, \)

and \( I_2 \text{ (max.)} = \frac{V_Y \text{ (max.)}}{R_2} + \frac{V_{REF}}{R_b} = 250 \mu A. \)

Resistor \( R_C \) supplies bias current for \( I_3 \) which allows the output to go negative.

Resistors \( R_{CX} \) and \( R_{CY} \) permit equation (6) to balance, i.e.:

\[
\begin{align*}
\left( \frac{V_X}{R_1} + \frac{V_{REF}}{R_a} \right) \frac{V_Y}{R_2} + \frac{V_{REF}}{R_b} &= \left( \frac{V_0}{R_0} + \frac{V_{REF}}{R_{CX}} \right) \frac{\frac{V_X}{R_2}}{R_{CY}} + \frac{V_{REF}}{R_D} \\
\frac{V_Y V_X}{R_1 R_2} + \frac{V_Y V_{REF} R_2}{R_a R_b} + \frac{V_0 V_{REF} R_2}{R_0 R_d} + \frac{V_{REF}^2}{R_{CX} R_{CY}} &= \frac{V_0 V_{REF}}{R_0 R_d} + \frac{V_Y V_{REF}}{R_{CX} R_{CY} d} + \frac{V_{REF}}{R_{D} R_d}
\end{align*}
\]
Cross-Product Cancellation
Cross-products are a result of the $V_X V_Y$ and $V_Y V_R$ terms. To the extend that $R_1 R_b = R_X R_d$, and $R_2 R_a = R_Y R_d$ cross-product cancellation will occur.

Arithmetic Offset Cancellation
The offset caused by the $V_{REF}^2$ term will cancel to the extent that $R_a R_b = R_0 R_d$, and the result is:

$$\frac{V_Y V_X}{R_1 R_2} = \frac{V_0 V_{REF}}{R_0 R_d} \text{ or } V_0 = V_X V_Y K$$

where $K = \frac{R_0 R_d}{V_{REF} R_1 R_2}$

Resistor Values
Inputs:

$V_X (\text{min.}) \leq V_X \leq V_X (\text{max.})$

$\Delta V_X = V_X (\text{max.}) - V_X (\text{min.})$

$V_Y (\text{min.}) \leq V_Y \leq V_Y (\text{max.})$

$\Delta V_Y = V_Y (\text{max.}) - V_Y (\text{min.})$

$V_{REF} = \text{Constant (+7V to +18V)}$

$K = \frac{V_0}{V_X V_Y}$ (Design Requirements)

$$R_1 = \frac{\Delta V_X}{200 \mu A}, \quad R_2 = \frac{\Delta V_Y}{200 \mu A}, \quad R_d = \frac{V_{REF}}{250 \mu A}$$

$$R_a = \frac{\Delta V_X V_{REF}}{250 \mu A \Delta V_X - 200 \mu A V_X (\text{max.})}$$

$$R_b = \frac{\Delta V_X V_{REF}}{250 \mu A \Delta V_Y - 200 \mu A V_Y (\text{max.})}$$

$$R_c = \frac{R_a R_b}{R_d}, \quad R_{CX} = \frac{R_1 R_b}{R_d}, \quad R_{cy} = \frac{R_c R_a}{R_d}$$

$$R_0 = \frac{\Delta V_X \Delta V_Y K}{160 \mu A}$$
Multiplying Circuit Offset Adjust

10K \leq R_5 = R_9 = R_{16} \leq 50K
R_7 = R_{11} = R_{14} = 100\Omega
R_6 = R_{10} = 100\Omega (V_S/0.05)
R_{15} = 100\Omega (V_S/0.10)
R_8 = R_1 \parallel R_a
R_{12} = R_2 \parallel R_b
R_{13} = R_0 \parallel R_C \parallel R_C X \parallel R_C Y

Procedure

1. Set all trimmer pots to 0V on the wiper.

2. Connect \( V_X \) input to ground. Put in a full scale square wave on \( V_Y \) input. Adjust \( XOS(R_5) \) for no square wave on \( V_O \) output (adjust for 0 feedthrough).

3. Connect \( V_Y \) input to ground. Put in a full scale square wave on \( V_X \) input. Adjust \( YOS(R_9) \) for no square wave on \( V_O \) output (adjust for 0 feedthrough).

4. Connect \( V_X \) and \( V_Y \) to ground. Adjust \( VOS(R_{16}) \) for 0V on \( V_O \) output.
**Extended Range Divider**

As with the extended range multiplier, resistors $R_{az}$ and $R_{ao}$ are added to cancel the cross-product error caused by the biasing resistors, i.e.

$$\left(\frac{V_X \cdot V_0 \cdot V_Z}{R_1 \cdot R_{ao} \cdot R_{az}} + \frac{V_{REF}}{R_b}\right) \left(\frac{V_{REF}}{R_c} \cdot \frac{V_{REF}}{R_d}\right) = \left(\frac{V_0 \cdot V_{REF}}{R_0} \cdot \frac{V_{REF}}{R_c} \cdot \frac{V_{REF}}{R_d}\right)$$

$$\frac{V_X \cdot V_{REF} \cdot V_0 \cdot V_{REF} \cdot V_Z \cdot V_{REF} \cdot V_{REF}^2}{R_1 \cdot R_{ao} \cdot R_{az} \cdot R_d \cdot R_{ao} \cdot R_d} = -\frac{V_0 \cdot V_{REF} \cdot V_{REF} \cdot V_{REF}^2}{R_0 \cdot R_{az} \cdot R_d \cdot R_{az} \cdot R_d}$$

To cancel cross-product and arithmetic offset:

$$R_{ao}R_b = R_0R_d, \ R_{az}R_b = R_4R_c \ and \ R_{a}R_b = R_cR_d$$

and the result is:

$$\frac{V_X \cdot V_{REF}}{R_1 \cdot R_b} = \frac{V_0 \cdot V_Z}{R_0 \cdot R_4} \ or \ V_0 = \frac{V_X}{V_Z K}$$

where $K = \frac{R_0R_d}{R_1R_b}$

**Note:** It is necessary to match the above resistor cross-products to within the amount of error tolerable in the output offset, i.e., with a 10V F.S. output, 0.1% resistor cross-product match will give 0.1% x 10V. Untrimmable output offset voltage.

**Resistor Values**

**Inputs:**

- $V_X(\text{min.}) \leq V_X \leq V_X(\text{max.})$
- $\Delta V_X = V_X(\text{max.}) - V_X(\text{min.})$
- $V_Z(\text{min.}) \leq V_Z \leq V_Z(\text{max.})$
- $\Delta V_Z = V_Z(\text{max.}) - V_Z(\text{min.})$
- $V_{REF} = \text{Constant (+7V to +18V)}$

**Outputs:**

- $V_0(\text{min.}) \leq V_0 \leq V_0(\text{max.})$
- $\Delta V_0 = V_0(\text{max.}) - V_0(\text{min.})$
- $K = \frac{V_0 \cdot V_Z}{V_X}$ (Design Requirement)

$$R_0 = \frac{\Delta V_0 \cdot V_{REF}}{750 \mu A}, \ R_b = \frac{\Delta V_{REF}}{250 \mu A}, \ R_4 = \frac{\Delta V_Z}{200 \mu A}$$

$$R_c = \frac{\Delta V_0 \cdot V_{REF}}{750 \mu A \Delta V_0 - 700 \mu A \ V_0(\text{max.})}$$

$$R_d = \frac{\Delta V_X \cdot V_{REF}}{250 \mu A \Delta V_Z - 200 \mu A \ V_Z(\text{max.})}$$

$$R_a = \frac{R_cR_d}{R_b}, \ R_{az} = \frac{R_cR_d}{R_b}, \ R_{ao} = \frac{R_b}{R_b}$$

$$R_1 = \frac{\Delta V_0 \Delta V_Z}{600 \mu A K}$$
Divider Circuit with Offset Adjustment

R18-R21 can be used in place of R9 to help cancel gain error due to resistor product mis-match (See Appendix 1).

General

10K ≤ R5 = R13 = R17 ≤ 50K
R7 + R8 = R1 + R2 + R6 + R8 + R10 + R12
R6 = R7 (Vs/0.05)
R9 = Rb
R10 = 100 x R4
R11 = 20K
R12 = 100K
R14 + R15 = R10 + R11 + R12
R16 = R15 (Vs/0.10)

Example: Two-Quad Divider

V0 = k(Vx/Vz), K = k, VREF = +Vs = +15V
-10 ≤ Vx ≤ +10, therefore ΔVx = 20
0 ≤ Vz ≤ +10, therefore ΔVz = 20
-10 ≤ V0 ≤ +10, therefore ΔV0 = 20
R0 = 26.7K  R1 = 333K
Rb = 60K  Rs, Rs17 = 10K
R4 = 50K  R7, R15 = 1K
Rc = 37.5K  R8, R11 = 20K
Rd = 300K  R6, R9, R16 = 300K
Rd = 187.5K  R10 = 4.7M
Rao = 31.25  R12 = 100K
Rao = 133K

Figure 7.
Divide Circuit Offset Adjustment Procedure

1. Set each trimmer pot to 0V on the wiper.

2. Connect $V_X$ (input) to ground. Put a DC voltage of approximately 1/2 $V_Z$ (max.) DC on the $V_Z$ (input) with an AC (squarewave is easiest) voltage of 1/2 $V_Z$ (max.) peak-to-peak superimposed on it. Adjust $XOS$ (R5) for zero feedthrough. (No AC at $V_0$)

   $V_Z$ (Max.)
   1/2 $V_Z$ (Max.)
   0V

3. Connect $V_X$ (input) to $V_Z$ (input) and put in the 1/2 $V_Z$(max.) DC with an AC of approximately 20 mV less than $V_Z$(max.).

   Adjust $XOS$ (R13) for zero feedthrough.

   $V_Z$ (Max.)
   1/2 $V_Z$ (Max.)
   0V

   $\leq 10$ mV

4. Return $V_X$ (input) to ground and connect $V_Z$(max.) DC on $V_Z$(input). Adjust output $V_{OS}(R17)$ for $V_0 = 0V$

5. Connect $V_X$ (input) to $V_Z$ (input) and and in $V_Z$ (max.) DC. (The output will equal K.) Decrease the input slowly until the output ($V_0 - K$) deviates beyond the desired accuracy. Adjust $XOS$ to bring it back into tolerance and return to Step 4. Continue steps 4 and 5 until $V_Z$ reduces to the lowest value desired.

Note: As the input to $V_X$ and $V_Z$ gets closer to zero (an illegal state) the system noise will predominate so much that an integrating voltmeter will be very helpful.

Square Root Circuit

$V_0 = N\sqrt{V_X}$

$\frac{V_X}{V_{REF}} \cdot \frac{V_{REF}}{R_1} \cdot \frac{V_{REF}}{R_4} \cdot \frac{V_0}{V_{REF}} \cdot \frac{V_{REF}}{R_0} \cdot \frac{R_0}{R_d}$

If $R_4 = R_0$, then $V_0 = V_X$ where $K = \frac{V_{REF}}{R_1}$

and $V_0 = N\sqrt{V_X}$ where $N = \sqrt{K}$

$0 \leq V_X \leq V_X$(max.) and $V_0$(max.) = $N\sqrt{V_X}$(max.)

$N = \frac{V_0}{\sqrt{V_X}}$ (Design Requirements)

$R_1 = \frac{V_0}{\sqrt{V_X}}^2$ (4.7kΩ)

$R_a = R_b = \frac{V_{REF}}{50\mu A}$

$R_c = \frac{V_{REF}}{150\mu A}$

$R_d = \frac{V_0}{50\mu A}$

$R_{ao} = \frac{V_0}{125\mu A}$

$R_0 = \frac{V_0}{225\mu A}$

Figure 8.
Square Root Circuit Offset Adjust

R14-R17 can be used in place of R9 to help reduce linearity error due to resistor product mis-match (See Appendix 1).

\[ 10 \text{K} \leq R_s = R_{13} \leq 50 \text{K} \]

\[ R_7 = 100 \Omega \]

\[ R_6 = R_7 \frac{V_s}{0.05} \]

\[ R_8 = R_1 \parallel R_a \parallel R_{30} \]

\[ R_b = R_b \]

\[ R_{10} = R_v \parallel R_C \]

\[ R_{11} = 100 \Omega \]

\[ R_{12} = R_{11} \frac{V_s}{0.1} \]

**Procedure**

1. Set both trimmer pots to 0V on the wiper.

2. Put in a full scale (0 to Vx(max.)) squarewave on Vx input. Adjust Xos(R5) for proper peak-to-peak amplitude on V0 output. (Scaling adjust)

3. Connect Vx input to ground. Adjust Vos(R13) for 0V on V0 output.

*Figure 9.*
Squaring Circuits \( V_0 = K V_X^2 \)

\[
\frac{V_X}{R_1} + \frac{2V_X V_{\text{REF}}}{R_1 R_a} + \frac{V_{\text{REF}}^2}{R_a^2} = \frac{V_0 V_{\text{REF}}}{R_0 R_d} + \frac{V_{\text{REF}}^2}{R_c R_d} + \frac{V_X V_{\text{REF}}}{R_c R_d}
\]

if \( R_a^2 = R_c R_d \) and \( R_1 R_a = 2 R_{\text{CX}} R_d \)

then \( \frac{V_0 V_{\text{REF}}}{R_0 R_d} = \frac{V_X^2}{R_1^2} \) or \( V_0 = K V_X^2 \) where \( K = \frac{R_0 R_d}{V_{\text{REF}} R_1^2} \)

\( V_X(\text{min.}) \leq V_X \leq V_X(\text{max.}) \quad \Delta V_X = V_X(\text{max.}) - V_X(\text{min.}) \)

\[
K = \frac{V_0}{V_X^2} \quad \text{(Design Requirement)}
\]

\[
R_1 = \frac{\Delta V_X}{200 \mu A}
\]

\[
R_a = \frac{\Delta V_X V_{\text{REF}}}{250 \mu A \Delta V_X - 200 \mu A V_X(\text{max.})}
\]

\[
R_d = \frac{V_{\text{REF}}}{250 \mu A}
\]

\[
R_c = \frac{R_a}{R_d}
\]

\[
R_{\text{cx}} = \frac{R_1 R_a}{2 R_d}
\]

\[
R_0 = \frac{\Delta V_X^2 K}{160 \mu A}
\]

Figure 10.
Squaring Circuits Offset Adjust

10K ≤ R_{10} = R_{11} ≤ 50K

R_8, R_{15} = 100Ω

R_9, R_{14} = \frac{V_S}{0.1} \Omega

R_s, R_6 = R_1 \parallel R_a

R_{16} = R_9 \parallel R_c \parallel R_a

Procedure
1. Set both trimmer pots to 0V on the wiper.
2. Put in a full scale (±Vx) squarewave on Vx input. Adjust ZOS(R10) for uniform output.
3. Connect Vx input to ground. Adjust VOS(R11) for 0V on V0 outputs.

Figure 11.
Appendix 1—System Errors

There are four types of accuracy errors which affect overall system performance. They are:

1. Nonlinearity—Incremental deviation from absolute accuracy.\(^{(1)}\)
2. Scaling Error—Linear deviation from absolute accuracy.
3. Output Offset—Constant deviation from absolute accuracy.
4. Feedthrough\(^{(2)}\)—Cross-product errors caused by input offsets and external circuit limitations.

This nonlinearity error in the transfer function of the 4200 is ±0.1% max. (±0.03 max. for 4200A).

\[ i.e., l_i = \frac{1}{i_d} = \pm0.1\% \text{ F.S.} \] \(^{(4)}\)

The other system errors are caused by voltage offsets on the inputs of the 4200 and can be as high as ±3.0% (±2.0% for 4200A).

\[ i.e., V_0 = \frac{V_XV_Y}{V_Z} \left( \frac{R_0R_4}{R_1R_2} \right) \pm3.0\% \text{ F.S.} \] \(3\) \(^{(4)}\)

Errors Caused by Input Offsets

\[ V_0 = \frac{R_0R_4}{R_0R_4} \left( \frac{V_XV_Y}{V_Z} \right) \pm \frac{1}{V_Z} \left( V_XV_{OSX} \pm V_YV_{OSY} \pm V_{OSZ} \pm V_{OSXV_{OSY}} \right) \]

- \(V_Y\) Feedthrough
- \(V_X\) Feedthrough
- Scaling Error
- Output Offset Error

System errors can be greatly reduced by externally trimming the input offset voltages of the 4200. (±3.0% F.S. for 4200 and ±0.1% for 4200A.)

![Figure 12. RC4200 Multiplier](image)

If \(XOS = XOSX, YOS = YOSY, ZOS = -VOSZ\),

then \(V_0 = \frac{V_XV_Y}{V_Z} \left( \frac{R_0R_4}{R_1R_2} \right) \pm0.3\% \text{ F.S.} \) \(3\)

**Figure 13. RC4200 with Input Offset Adjustment**

Extended Range Circuit Errors

The extended range configurations have a disadvantage in that additional accuracy errors may be introduced by resistor product mismatching.

Multiplier (Figure 6)

An error in resistor product matching will cause an equivalent feedthrough or output offset error:

1. \(R_1R_b = RCXRd \pm \alpha, V_X\) feedthrough \((V_Y = 0) = \alpha V_X\)
2. \(R_2R_a = RCYRd \pm \beta, V_Y\) feedthrough \((V_X = 0) = -\beta V_Y\)
3. \(R_4R_b = RCYd \pm \gamma, V_0\) offset \((V_X = V_Y = 0) = \pm\gamma VR\) **

*Output offset errors can always be trimmed out with the output op amp offset adjust, \(VOS\) (R16).
Reducing Mismatch Errors (Figure 4)

You need not use .01% resistors to reduce resistor product mismatch errors. Here are a couple of ways to squeeze maximum accuracy out of the extended range multiplier (see Figure 4) using 1% resistors.

Method #1

VX feedthrough, for example, occurs when VY = 0 and VOSY ≠ 0. This VX feedthrough will equal ±VX VOSY. Also, if VOSZ ≠ 0, there is a VX feedthrough equal to VX VOSZ. A resistor-product error of α will cause a VX feedthrough of ±α VX. Likewise, VY feedthrough errors are: ±VY VOSX, ±VY VOSZ and ±β VY

Total feedthrough:
±VX VOSY ±VY VOSX ±α VX ±β VY ±(VX + VY) VOSZ

By carefully abusing XOS(R5), YOS(R9) and ZOS(R20) this equation can be made to very nearly equal zero and the feedthrough error will practically disappear.

A residual of set will probably remain which can be trimmed out with VOS(R16) at the output of amp.

Method #2

Notice that the ratios of R1 Rb; RCX Rd and R2 Rb; RCY Rd are both dependent of Rd also that R1, R2, Ra and Rb are all functions of the maximum input requirements. By designing a multiplier for the same input ranges on both VX and VY then R1 = R2, RCX = RCY and Ra = Rb. (Note: it is acceptable to design a four quadrant multiplier and use only two quadrants of it.)

Select Rd to be 1% or 2% below (or above) the calculated value. This will cause α and β to both be positive (or negative) by nearly the same amount. Now the effective value of Rd can be trimmed with an offset adjustment ZOS(R20) on pin 5.

This technique will cause: 1) a slight gain error which can be compensated for with the R0 value, and 2) an output of offset error that can be trimmed out with VOS(R16) on the output op amp.

Extended Range Divider (Figure 6)

The only cross-product error of interest is the VZ feedthrough (VX = 0 and VOSX ≠ 0) which is easily adjusted with XOS(R5).

Resistor product mismatch will cause scaling errors (gain) that could be a problem for very low values of VZ. Adjustments to YOS(R18) can be made to improve the high gain accuracy.

Square Root and Squaring (Figures 9 and 11)

These circuits are functions of single variables so feedthrough, as such, is not a consideration. Cross product errors will affect incremental accuracy that can be corrected YOS(R14) or ZOS(R10).
Appendix 2—Applications

Design Considerations for RMS-to-DC Circuits

Average Value
Consider $V_{IN} = A \sin \omega t$. By definition,

$$ V_{AG} = \frac{1}{T} \int_{0}^{T} V_{IN} dt $$

Where $T = \text{Period}$

$$ \omega = 2\pi f = \frac{2\pi}{T} $$

$$ V_{AG} = \frac{2}{\pi} \int_{0}^{T/2} A \sin \omega t dt $$

$$ = \frac{2A}{\pi} \left[ -\cos(\pi) + \cos(0) \right] $$

Average Value of $A \sin \omega t$ is $\frac{2A}{\pi}$

RMS Value
Again, consider $V_{IN} = A \sin \omega t$

$$ V_{rms} = \sqrt{\frac{1}{T} \int_{0}^{T} [V_{IN}]^2 dt} $$

$V_{rms}$ for $A \sin \omega t$:

$$ V_{rms} = \sqrt{\frac{1}{T} \int_{0}^{T} A^2 \sin^2 \omega t dt} $$

$$ V_{rms} = \sqrt{\frac{A^2}{T} \left[ \frac{T}{2} \left( 1 - \frac{1}{2} \cos 2\omega t \right) \right]} $$

$$ V_{rms} = \sqrt{\frac{A^2}{2} \left[ \frac{1}{2} - \frac{1}{4\omega} \sin 2\omega t \right]} $$

$$ V_{rms} = \frac{A}{\sqrt{2}} $$

therefore, the rms value of $A \sin \omega t$ becomes:

$$ V_{rms} = \frac{A}{\sqrt{2}} $$

RMS Value for Rectified Sine Waves
Consider $V_{IN} = |A \sin \omega t|$, a rectified wave. To solve, integrate of each half cycle.

i.e. $\frac{1}{T} \int_{0}^{T} V_{IN}^2 dt = $ $\frac{1}{T} \int_{0}^{T/2} A^2 \sin^2 \omega t dt + \frac{T}{2} (-A \sin \omega t)^2 dt$

This is the same as $\frac{1}{T} \int_{0}^{T} A^2 \sin^2 \omega t dt$

so, $|A \sin \omega t|_{rms} = A \sin \omega t_{rms}$

Practical Consideration: $|A \sin \omega t|$ has high-order harmonics; $A \sin \omega t$ does not. Therefore, non-ideal integrators may cause different errors for two approaches.

Figure 14.
Amplitude Modulator with A.G.C. (Figure 16)

In many AC modulator applications, unwanted output modulation is caused by variations in carrier input amplitude. The versatility of the RC4200 multiplier can be utilized to eliminate this undesired fluctuation. The extended range multiplier circuit (Figure 4) shows an output amplitude inversely proportional to the reference voltage \( V_{REF} \).

i.e., \( V_0 = \frac{V_X V_Y}{V_{REF}} \frac{R_0 R_d}{R_1 R_2} \)

By making \( V_{REF} \) proportional to \( V_Y \) (where \( V_Y \) is the carrier input) such that:

\[ V_{REF} = V_H = \int (|V_Y|) \]

Then the denominator becomes a variable value that automatically provides constant gain, such that the modulating input \( V_X \) modulates the carrier \( V_Y \) with a fixed scale factor even though the carrier varies in amplitude.

If \( V_H \) is made proportional to the average value of \( A \) (i.e., \( 2A/\pi \)) and scaled by a value of \( \pi/2 \) then:

\[ V_H = A \]

and if: \( V_X \) = Modulating input \( (V_M) \)

and: \( V_Y \) Carrier input \( (A) \)

Then: \( V_0 = K V_M \sin \vartheta \) where \( K = \frac{R_0 R_d}{R_1 R_2} \)

The resistor scaling is determined by the dynamic range of the carrier variation and modulating input.

The resistor values are solved, as with the other extended range circuits, in terms of the input voltages.

Input voltages:
- Modulation voltage \( (V_M) \): \( 0 \leq V_M \leq V_X(\text{max}) \)
- Carrier \( (V_Y) \): \( V_Y = A \sin \vartheta \)
- Carrier amplitude fluctuation \( (\Delta A) \):
  - \( A(\text{min}) \) \( \leq V_Y \leq A(\text{max}) \) \( \sin \vartheta \text{at} \)
- Dynamic Range \( (N) \):
  - \( A(\text{max})/A(\text{min}) \)
  - \( A(\text{max}) = V_H(\text{max}) \) and \( A(\text{min}) = V_H(\text{min}) \)
The maximum and minimum values for \( I_1 \) and \( I_2 \) lead to:

\[
I_1(\text{max.}) = \frac{V_X(\text{max.})}{R_1} + \frac{V_H(\text{max.})}{R_a} = 250\mu A
\]

\[
I_1(\text{min.}) = \frac{V_H(\text{min.})}{R_a} = 50\mu A \quad V_M(\text{min.}) = 0
\]

\[
I_2(\text{max.}) = \frac{A(\text{max.})}{R_2} + \frac{V_H(\text{max.})}{R_a} = 250\mu A
\]

\[
I_2(\text{min.}) = \frac{V_H(\text{min.})}{R_a} = 50\mu A
\]

For a dynamic range of \( N \), where

\[
N = \frac{A(\text{max.})}{A(\text{min.})} < 5,
\]

These equations combine to yield:

\[
R_1 = \frac{V_X(\text{max.})}{(5 - N) 50\mu A}, \quad R_2 = \frac{A(\text{max.})}{(5 - N) 50\mu A},
\]

\[
R_a = \frac{A(\text{min.})}{50\mu A} \quad \text{and} \quad R_0 = K \frac{R_1 R_2}{R_a}
\]

**Example #1**

\( V_Y = A \sin \omega t \), \( 2.5V \leq A \leq 10V \), therefore \( N = 4 \)

\( 0V \leq V_M \leq 10V \), therefore \( V_X(\text{max.}) = 10V \)

\( K = 1 \), therefore \( V_0 = V_M \sin \omega t \)

\[
R_1 = \frac{V_X(\text{max.})}{50\mu A} = \frac{10V}{50\mu A} = 200K
\]

\[
R_1 = \frac{A(\text{max.})}{50\mu A} = \frac{10V}{50\mu A} = 200K
\]

\[
R_a = \frac{A(\text{min.})}{50\mu A} = \frac{2.5V}{50\mu A} = 50K
\]

\[
R_0 = K \frac{R_1 R_2}{R_a} = 1 \times 200K \times 200K = 800K
\]

**Example #2**

\( V_Y = A \sin \omega t \), \( 3 \leq A \leq 6 \), therefore \( N = 2 \)

\( 0V \leq V_M \leq 8V \), therefore \( V_X(\text{max.}) = 8V \)

\( K = .2 \), therefore \( V_0 = .2 V_M \sin \omega t \)

so:

\[
R_1 = 53.3K, \quad R_2 = 40K
\]

\( R_a = 60K \) and \( R_0 = 7.11K \)
Limited Range, First Quadrant Applications

The following circuit has the advantage that cross-product errors are due only to input offsets and nonlinearity error is slightly less for lower input currents.

The circuit also has no standby current to add to the noise content, although the signal-to-noise ratio worsens at very low input currents (1-5 μA) due to the noise current of the input stages.

The RsCs filter circuits are added to each input to improve the stability for input currents below 50 μA.

Caution

The bandpass drops off significantly for lower currents (<50 μA) and non-symmetrical rise and fall times can cause second harmonic distortion.

Thermal Symmetry

The scale factor is sensitive to temperature gradients across the chip in the lateral direction. Where possible, the package should be oriented such that forces generating temperature gradients are located physically on the line of thermal symmetry. This will minimize scale-factor error due to thermal gradients.
Figure 18.
Figure 19a. Output Noise Current (b) vs. Input Currents \((I_1, I_2)\) for \(I_4 = 250\mu A\)

Figure 19b. Output Noise Current (b) vs. Input Currents \((I_4, I_1)\) for \(I_2 = 250\mu A\)

Figure 20. AC Feedthrough vs. Frequency

Multiplier Configuration

\[ V_o = \frac{V_x V_y}{10} \]

\[ V_x = 0 \quad V_y = 10 \sin \omega t \]

\[ V_x = 0 \quad V_y = 10 \sin \omega t \]
### Ordering Information

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<thead>
<tr>
<th>Part Number</th>
<th>Package</th>
<th>Operating Temperature Range</th>
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<td>RC4200N</td>
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<td>0°C to +70°C</td>
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<tr>
<td>RC4200AN</td>
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<td>RM4200D</td>
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<tr>
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<tr>
<td>RM4200AD/883B</td>
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<td>-55°C to +125°C</td>
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**Notes:**

/883B suffix denotes MIL-STD-883, Level B processing

N = 8-Lead Plastic DIP

D = 8-Lead Ceramic DIP